1.

(a)

Suppose n(n-1)/2≤cn2 , and c=1; then n2+n≥0 if n≥1, so n0=1, then n(n-1)/2 is O(n2).

(b)

Suppose max(n3, 10n2)=n3 , then n3≥10n2 , so n≥ 10, then for c=1, n3 ≤n3 , so for c=1, n0 =10, max(n3, 10n2) is O(n3)

(c)

Since the sum (nk + nk + … + nk) is greater than or equal to the sum (1k + 2k + … + nk) for n≥1 and k is a integer, and (nk + nk + … + nk) =(nk+1) , so (1k + 2k + … + nk) ≤ (nk+1) for n≥1 , which means (1k + 2k + … + nk) is O(nk+1) for c=1 and n0=1

(d)

Assume that p(n) = ank+bnk-1+…+zn

Because ank+bnk-1+…+zn ≤ ank+bnk+…+znk  for n ≥ 1

Which equals to ank+bnk-1+…+zn ≤ (a+b+…+z) nk

So p(n) is O(nk) when c= sum of the coefficient and n0=1

2.

a) (log n)n is faster.

b) (log(n))k is faster

c) (log n)! is faster.

d) nn is faster.

3.

From the given conditions, we know that there are constants c1, c2, n1 and n2 such that

f1(n) ≤ c1\* g1(n) for all n ≥ n1

f2(n) ≤ c2\* g2(n) for all n ≥ n2.

Let c = max(c1, c2) and n0 = max(n1, n2).

Then, f1(n) + f2(n) ≤ c1\* g1(n) + c2\* g2(n) for all n ≥ n0

≤ c\* ( g1(n) + g2(n) ) for all n ≥ n0.

f1(n) + f2(n) ≤ c\* ( 2\*max( g1(n), g2(n) ) ) for all n ≥ n0.

≤ 2c\* ( max( g1(n), g2(n) ) ) for all n ≥ n0.

Then f1(n) + f2(n) is O( max( g1(n), g2(n) ) )

4. Suppose that it were true. Then there exists constants c and n0 such that n ≤ cn/2 for all n≥n0, which equals to c≥2 for n>0. So as long as c is a constant not smaller than 2 and n>0,

n is O(n/2 ).

5.

FALSE. Suppose that it were true. Then there exists constants c and n0 such that 3 n ≤ c2 n for all n ≥ n0. The last requirement is equivalent to (3/2)n ≤ c for all n ≥ n0. However, (3/2)n → ∞ as n → ∞, so (3/2)n ≤ c cannot be true for all n ≥ n0 for any constant c